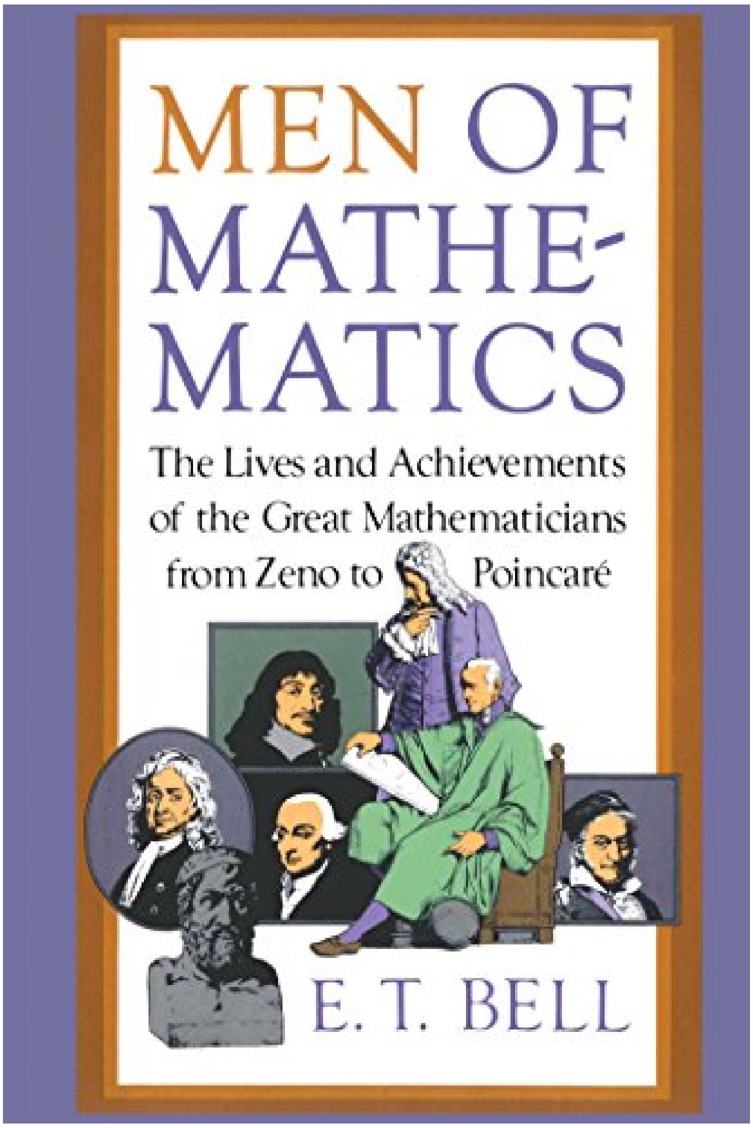
Men of Mathematics (Touchstone Book) (English Edition)



Par E.T. Bell
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Par E.T. Bell : Men of Mathematics (Touchstone Book) (English Edition) before purchasing it in order to gage whether or not it would be worth my time, and all praised Men of Mathematics (Touchstone Book) (English Edition):

Description :

Prsentation de l'diteurFrom one of the greatest minds in contemporary mathematics, Professor E.T. Bell, comes a witty, accessible, and fascinating look at the beautiful craft and enthralling history of mathematics.Men of Mathematics provides a rich account of major mathematical milestones, from the geometry of the Greeks through Newtons calculus, and on to the laws of probability, symbolic logic, and the fourth dimension. Bell breaks down this majestic history of ideas into a series of engrossing biographies of the great mathematicians who made progress possibleand who also led intriguing, complicated, and often surprisingly entertaining lives. Never pedantic or dense, Bell writes with clarity and simplicity to distill great mathematical concepts into their most understandable forms for the curious everyday reader. Anyone with an interest in math may learn from these rich lessons, an advanced degree or extensive research is never necessary.ExtraitCHAPTER ONEIntroductionThis section is headed Introduction rather than Preface (which it really is) in the hope of decoying habitual preface-skippers into reading -- for their own comfort -- at least the following paragraphs down to the first row of stars before going on to meet some of the great mathematicians. I should like to emphasize first that this book is not intended, in any sense, to be a history of mathematics, or any section of such a history.The lives of mathematicians presented here are addressed to the general reader and to others who may wish to see what sort of human beings the men were who created modern mathematics. Our object is to lead up to some of the dominating ideas governing vast tracts of mathematics as it exists today and to do this through the lives of the men responsible for those ideas.Two criteria have been applied in selecting names for inclusion: the importance for modern mathematics of a man's work; the human appeal of the man's life and character. Some qualify under both heads, for example Pascal, Abel, and Galois; others, like Gauss and Cayley, chiefly under the first, although both had interesting lives. When these criteria clash or overlap in the case of several claimants to remembrance for a particular advance, the second has been given precedence as we are primarily interested here in mathematicians as human beings.Of recent years there has been a tremendous surge of general interest in science, particularly physical science, and its bearing on our rapidly changing philosophical outlook on the universe. Numerous excellent accounts of current advances in science, written in as un-technical language as possible, have served to lessen the gap between the professional scientist and those who must make their livings at something other than science. In many of these expositions, especially those concerned with relativity and the modern quantum theory, names occur with which the general reader cannot be expected to be familiar -- Gauss, Cayley, Riemann, and Hermite, for instance. With a knowledge of who these men were, their part in preparing for the explosive growth of physical science since 1900, and an appreciation of their rich personalities, the magnificent achievements of science fall into a truer perspective and take on a new significance.The great mathematicians have played a part in the evolution of scientific and philosophic thought comparable to that of the philosophers and scientists themselves. To portray the leading features of that part through the lives of master mathematicians, presented against a background of some of the dominant problems of their times, is the purpose of the following chapters. The emphasis is wholly on modern mathematics, that is, on those great and simple guiding ideas of mathematical thought that are still of vital importance in living, creative science and mathematics.It must not be imagined that the sole function of mathematics -- "the handmaiden of the sciences" -- is to serve science. Mathematics has also been called "the Queen of the Sciences." If occasionally the Queen has seemed to beg from the sciences she has been a very proud sort of beggar, neither asking nor accepting favors from any of her more affluent sister sciences. What she gets she pays for. Mathematics has a light and wisdom of its own, above any possible application to science, and it will richly reward any intelligent human being to catch a glimpse of what mathematics means to itself. This is not the old doctrine of art for art's sake; it is art for humanity's sake. After all, the whole purpose of science is not technology -- God knows we have gadgets enough already; science also explores depths of a universe that will never, by any stretch of the imagination, be visited by human beings or affect our material existence. So we shall attend also to some of the things which the great mathematicians have considered worthy of loving understanding for their intrinsic beauty.Plato is said to have inscribed "Let no man ignorant of geometry enter here" above the entrance to his Academy. No similar warning need be posted here, but a word of advice may save some overconscientious reader unnecessary anguish. The gist of the story is in the lives and personalities of the creators of modern mathematics, not in the handful of formulas and diagrams scattered through the text. The basic ideas of modern mathematics, from which the whole vast and intricate complexity has been woven by thousands of workers, are simple, of boundless scope, and well within the understanding of any human being with normal intelligence. Lagrange (whom we shall meet later) believed that a mathematician has not thoroughly understood his own work till he has made it so clear that he can go out and explain it effectively to the first man he meets on the street.This of course is an ideal and not always attainable. But it may be recalled that only a few years before Lagrange said this the Newtonian "law" of gravitation was an incomprehensible mystery to even highly educated persons. Yesterday the Newtonian "law" was a commonplace which every educated person accepted as simple and true; today Einstein's relativistic theory of gravitation is where Newton's "law" was in the early decades of the eighteenth century; to-morrow or the day after Einstein's theory will seem as "natural" as Newton's "law" seemed yesterday. With the help of time Lagrange's ideal is not unattainable.Another great French mathematician, conscious of his own difficulties no less than his readers', counselled the conscientious not to linger too long over anything hard but to "Go on, and faith will come to you." In brief, if occasionally a formula, a diagram, or a paragraph seems too technical, skip it. There is ample in what remains.Students of mathematics are familiar with the phenomenon of "slow development," or subconscious assimilation: the first time something new is studied the details seem too numerous and hopelessly confused, and no coherent impression of the whole is left on the mind. Then, on returning after a rest, it is found that everything has fallen into place with its proper emphasis -- like the development of a photographic film. The majority of those who attack analytic geometry seriously for the first time experience something of the sort. The calculus on the other hand, with its aims clearly stated from the beginning, is usually grasped quickly. Even professional mathematicians often skim the work of others to gain a broad, comprehensive view of the whole before concentrating on the details of interest to them. Skipping is not a vice, as some of us were told by our puritan teachers, but a virtue of common sense.As to the amount of mathematical knowledge necessary to understand everything that some will wisely skip, I believe it may be said honestly that a high school course in mathematics is sufficient. Matters far beyond such a course are frequently mentioned, but wherever they are, enough description has been given to enable anyone with high school mathematics to follow. For some of the most important ideas discussed in connection with their originators -- groups, space of many dimensions, non-Euclidean geometry, and symbolic logic, for example -- less than a high school course is ample for an understanding of the basic concepts. All that is needed is interest and an undistracted head. Assimilation of some of these invigorating ideas of modem mathematical thought will be found as refreshing as a drink of cold water on a hot day and as inspiring as any art.To facilitate the reading, important definitions have been repeated where necessary, and frequent references to earlier chapters have been included from time to time.The chapters need not be read consecutively. In fact, those with a speculative or philosophical turn of mind may prefer to read the last chapter first. With a few trivial displacements to fit the social background the chapters follow the chronological order.It would be impossible to describe all the work of even the least prolific of the men considered, nor would it be profitable in an account for the general reader to attempt to do so. Moreover, much of the work of even the greater mathematicians of the past is now of only historical interest, having been included in more general points of view. Accordingly only some of the conspicuously new things each man did are described, and these have been selected for their originality and importance in modern thought.Of the topics selected for description we may mention the following (among others) as likely to interest the general reader: the modem doctrine of the infinite (chapters 2, 29); the origin of mathematical probability (chapter 5); the concept and importance of a group (chapter 15); the meanings of invariance (chapter 21); non-Euclidean geometry (chapter 16 and part of 14); the origin of the mathematics of general relativity (last part of chapter 26); properties of the common whole numbers (chapter 4), and their modem generalization (chapter 25); the meaning and usefulness of so-called imaginary numbers -- like [underroot-1] (chapters 14, 19); symbolic reasoning (chapter 23). But anyone who wishes to get a glimpse of the power of the mathematical method, especially as applied to science, will be repaid by seeing what the calculus is about (chapters 2, 6).Modem mathematics began with two great advances, analytic geometry and the calculus. The former took definite shape in 1637, the latter about 1666, although it did not become public property till a decade later. Though the idea behind it all is childishly simple, yet the method of analytic geometry is so powerful that very ordinary boys of seventeen can use it to prove results which would have baffled the greatest of the Greek geometers -- Euclid, Archimedes, and Apollonius. The man, Descartes, who finally crystallized this great method had a particularly full and interesting life.In saying that Descartes was responsible for the creation of analytic geometry we do not mean to imply that the new method sprang full-armed from his mind alone. Many before him had made significant advances toward the new method, but it remained for Descartes to take the final step and actually to put out the method as a definitely workable engine of geometrical proof, discovery, and invention. But even Descartes must share the honor with Fermat.Similar remarks apply to most of the other advances of modem mathematics. A new concept may be "in the air" for generations until some one man -- occasionally two or three together -- sees clearly the essential detail that his predecessors missed, and the new thing comes into being. Relativity, for example, is sometimes said to have been the great invention reserved by time for the genius of Minkowski. The fact is, however, that Minkowski did not create the theory of relativity and that Einstein did. It seems rather meaningless to say that So-and-so might have done this or that if circumstances had been other than they were. Any one of us no doubt could jump over the moon if we and the physical universe were different from what we and it are, but the truth is that we do not make the jump.In other instances, however, the credit for some great advance is not always justly placed, and the man who first used a new method more powerfully than its inventor sometimes gets more than his due. This seems to be the case, for instance, in the highly important matter of the calculus. Archimedes had the fundamental notion of limiting sums from which the integral calculus springs, and he not only had the notion but showed that he could apply it. Archimedes also used the method of the differential calculus in one of his problems. As we approach Newton and Leibniz in the seventeenth century the history of the calculus becomes extremely involved. The new method was more than merely "in the air" before Newton and Leibniz brought it down to earth; Fermat actually had it. He also invented the method of Cartesian geometry independently of Descartes. In spite of indubitable facts such as these we shall follow tradition and ascribe to each great leader what a majority vote says he should have, even at the risk of giving him a little more than his just due. Priority after all gradually loses its irritating importance as we recede in time from the men to whom it was a hotly contested cause of verbal battles while they and their partisans lived.Those who have never known a professional mathematician may be rather surprised on meeting some, for mathematicians as a class are probably less familiar to the general reader than any other group of brain workers. The mathematician is a much rarer character in fiction than his cousin the scientist, and when he does appear in the pages of a novel or on the screen he is only too apt to be a slovenly dreamer totally devoid of common sense -- comic relief. What sort of mortal is he in real life? Only by seeing in detail what manner of men some of the great mathematicians were and what kind of lives they lived, can we recognize the ludicrous untruth of the traditional portrait of a mathematician.Strange as it may seem, not all of the great mathematicians have been professors in colleges or universities. Quite a few were soldiers by profession; others went into mathematics from theology, the law, and medicine, and one of the greatest was as crooked a diplomat as ever lied for the good of his country. A few have had no profession at all. Stranger yet, not all professors of mathematics have been mathematicians. But this should not surprise us when we think of the gulf between the average professor of poetry drawing a comfortable salary and the poet starving to death in his garret.The lives that follow will at least suggest that a mathematician can be as human as anybody else -- sometimes distressingly more so. In ordinary social contacts the majority have been normal. There have been eccentrics in mathematics, of course; but the percentage is no higher than in commerce or the professions. As a group the great mathematicians have been men of all-round ability, vigorous, alert, keenly interested in many things outside of mathematics and, in a fight, men with their full share of backbone. As a rule mathematicians have been bad customers to persecute; they have usually been capable of returning what they received with compound interest. For the rest they were geniuses of tremendous accomplishment marked off from the majority of their gifted fellowmen only by an irresistible impulse to do mathematics. On occasion mathematicians have been (and some still are in France) extremely able administrators.In their politics the great mathematicians have ranged over the whole spectrum from reactionary conservatism to radical liberalism. It is probably correct to say that as a class they have tended slightly to the left in their political opinions. Their religious beliefs have included everything from the narrowest orthodoxy -- sometimes shading into the blackest bigotry -- to complete skepticism. A few were dogmatic and positive in their assertions concerning things about which they knew nothing, but most have tended to echo the great Lagrange's "I do not know."Another characteristic calls for mention here, as several writers and artists (some from Hollywood) have asked that it be treated -- the sex life of great mathematicians. In particular these inquirers wish to know how many of the great mathematicians have been perverts -- a somewhat indelicate question, possibly, but legitimate enough to merit a serious answer in these times of preoccupation with such topics. None. Some lived celibate lives, usually on account of economic disabilities, but the majority were happily married and brought up their children in a civilized, intelligent manner. The children, it may be noted in passing, were often gifted far above the average. A few of the great mathematicians of bygone centuries kept mistresses when such was the fashionable custom of their times. The only mathematician discussed here whose life might offer something of interest to a Freudian is Pascal.Returning for a moment to the movie ideal of a mathematician, we note that sloppy clothes have not been the invariable attire of great mathematicians. All through the long history of mathematics about which we have fairly detailed knowledge, mathematicians have paid the same amount of attention to their personal appearance as any other equally numerous group of men. Some have been fops, others slovens; the majority, decently inconspicuous. If today some earnest individual affecting spectacular clothes, long hair, a black sombrero, or any other mark of exhibitionism, assures you that he is a mathematician, you may safely wager that he is a psychologist turned numerologist.The psychological peculiarities of great mathematicians is another topic in which there is considerable interest. Poincare will tell us something about the psychology of mathematical creation in a later chapter. But on the general question not much can be said till psychologists call a truce and agree among themselves as to what is what. On the whole the great mathematicians have lived richer, more virile lives than those that fall to the lot of the ordinary hard-working mortal. Nor has this richness been wholly on the side of intellectual adventuresomeness. Several of the greater mathematicians have had more than their share of physical danger and excitement, and some of them have been implacable haters -- or, what is ultimately the same, expert controversialists. Many have known the lust of battle in their prime, reprehensibly enough, no doubt, but still humanly enough, and in knowing it they have experienced something no jelly-fish has ever felt: "Damn braces, Bless relaxes," as that devout Christian William Blake put it in his Proverbs of Hell.This brings us to what at first sight (from the conduct of several of the men considered here) may seem like a significant trait of mathematicians -- their hair-trigger quarrelsomeness. Following the lives of several of these men we get the impression that a great mathematician is more likely than not to think others are stealing his work, or disparaging it, or not doing him sufficient honor, and to start a row to recover imaginary rights. Men who should have been above such brawls seem to have gone out of their way to court battles over priority in discovery and to accuse their competitors of plagiarism. We shall see enough dishonesty to discount the superstition that the pursuit of truth necessarily makes a man truthful, but we shall not find indubitable evidence that mathematics makes a man bad-tempered and quarrelsome.Another "psychological" detail of a similar sort is more disturbing. Envy is carried up to a higher level. Narrow nationalism and international jealousies, even in impersonal pure mathematics, have marred the history of discovery and invention to such an extent that it is almost impossible in some important instances to get at the facts or to form a just estimate of the significance of a particular man's work for modern thought. Racial fanaticism -- especially in recent times -- has also complicated the task of anyone who may attempt to give an unbiased account of the lives and work of scientific men outside his own race or nation.An impartial account of western mathematics, including the award to each man and to each nation of its just share in the intricate development, could be written only by a Chinese historian. He alone would have the patience and the detached cynicism necessary for disentangling the curiously perverted pattern to discover whatever truth may be concealed in our variegated occidental boasting.Even in restricting our attention to the modern phase of mathematics we are faced with a problem of selection that must be solved somehow. Before the solution adopted here is indicated it will be of interest to estimate the amount of labor that would be required for a detailed history of mathematics on a scale similar to that of a political history for any important epoch, say that of the French Revolution or the American Civil War.When we begin unravelling a particular thread in the history of mathematics we soon get a discouraged feeling that mathematics itself is like a vast necropolis to which constant additions are being made for the eternal preservation of the newly dead. The recent arrivals, like some of the few who were shelved for perpetual remembrance 5000 years ago, must be so displayed that they shall seem to retain the full vigor of the manhood in which they died; in fact the illusion must be created that they have not yet ceased living. And the deception must be so natural that even the most skeptical archaeologist prowling through the mausoleums shall be moved to exclaim with living mathematicians themselves that mathematical truths are immortal, imperishable; the same (yesterday, today, and forever; the very stuff of which eternal verities are fashioned and the one glimpse of changelessness behind all the recurrent cycles of birth, death, and decay our race has ever caught. Such may indeed be the fact; many, especially those of the older generation of mathematicians, hold it to be no less.But the mere spectator of mathematical history is soon over-whelmed by the appalling mass of mathematical inventions that still maintain their vitality and importance for modern work, as discoveries of the past in any other field of scientific endeavor do not, after centuries and tens of centuries.A span of less than a hundred years covers everything of significance in the French Revolution or the American Civil War, and less than five hundred leaders in either played parts sufficiently memorable to merit recording. But the army of those who have made at least one definite contribution to mathematics as we know it soon becomes a mob as we look back over history; 6000 or 8000 names press forward for some word from us to preserve them from oblivion, and once the bolder leaders have been recognized it becomes largely a matter of arbitrary, illogical legislation to judge who of the clamoring multitude shall be permitted to survive and who be condemned to be forgotten.This problem scarcely presents itself in describing the development of the physical sciences. They also reach far back into antiquity; yet for the most of them $50 years is a sufficient span to cover everything of importance to modern thought. But whoever attempts to do full, human justice to mathematics and mathematicians will have a wilderness of 6000 years in which to exercise such talents as he may have, with that mob of 6000 to 8000 claimants before him for discrimination and attempted justice.The problem becomes more desperate as we approach our own times. This is by no means due to our closer proximity to the men of the two centuries immediately preceding our own, but to the universally acknowledged fact (among professional mathematicians) that the nineteenth century, prolonged into the twentieth, was, and is, the greatest age of mathematics the world has ever known. Compared to what glorious Greece did in mathematics the nineteenth century is a bonfire beside a penny candle.What threads shall we follow to guide us through this labyrinth of mathematical inventions? The main thread has already been indicated: that which leads from the half-forgotten past to some of those dominating concepts which now govern boundless empires of mathematics-but which may themselves be dethroned tomorrow to make room for yet vaster generalizations. Following this main thread we shall pass by the developers in favor of the originators.Both inventors and perfectors are necessary to the progress of any science. Every explorer must have, in addition to his scouts, his followers to inform the world as to what he has discovered. But to the majority of human beings, whether justly or not is beside the point, the explorer who first shows the new way is the more arresting personality, even if he himself stumbles forward but half a step. We shall follow the originators in preference to the developers. Fortunately for historical justice the majority of the great originators in mathematics have also been peerless developers.Even with this restriction the path from the past to the present may not always be clear to those who have not already followed it. So we may state here briefly what the main guiding clue through the whole history of mathematics is.From the earliest times two opposing tendencies, sometimes helping one another, have governed the whole involved development of mathematics. Roughly these are the discrete and the continuous.The discrete struggles to describe all nature and all mathematics atomistically, in terms of distinct, recognizable individual elements, like the bricks in a wall, or the numbers 1,2,3,...The continuous seeks to apprehend natural phenomena -- the course of a planet in its orbit, the flow of a current of electricity, the rise and fall of the tides, and a multitude of other appearances which delude us into believing that we know nature -- in the mystical formula of Heraclitus: "All things flow." Today (as will be seen in the concluding chapter), "flow," or its equivalent, "continuity," is so unclear as to be almost devoid of meaning. However, let this pass for the moment.Intuitively we feel that we know what is meant by "continuous motion" -- as of a bird or a bullet through the air, or the fall of a raindrop. The motion is smooth; it does not proceed by jerks; it is unbroken. In continuous motion or, more generally, in the concept of continuity itself, the individualized numbers 1,2,3,..., are not the appropriate mathematical image. All the points on a segment of a straight line, for instance, have no such clear-cut individualities as have the numbers of the sequence 1,2,3,..., where the step from one member of the sequence to the next is the same (namely 1:1 + 2 = 3, 1 + 3 = 4, and so on); for between any two points on the line segment, no matter how close together the points may be, we can always find, or at least imagine, another point: there is no "shortest" step from one point to the "next." In fact there is no next point at all.The last -- the conception of continuity, "no nextness" -- when developed in the manner of Newton, Leibniz, and their successors leads out into the boundless domain of the calculus and its innumerable applications to science and technology, and to all that is today called mathematical analysis. The other, the discrete pattern based on 1,2,3,..., is the domain of algebra, the theory of numbers, and symbolic logic. Geometry partakes of both the continuous and the discrete.A major task of mathematics today is to harmonize the continuous and the discrete, to include them in one comprehensive mathematics, and to eliminate obscurity from both.It may be doing our predecessors an injustice to emphasize modern mathematical thought with but little reference to the pioneers who took the first and possibly the most difficult steps. But nearly everything useful that was done in mathematics before the seventeenth century has suffered one of two fates: either it has been so greatly simplified that it is now part of every regular school course, or it was long since absorbed as a detail in work of greater generality.Things that now seem as simple as common sense -- our way of writing numbers, for instance, with its "place system" of value and the introduction of a symbol for zero, which put the essential finishing touch to the place system -- cost incredible labor to invent. Even simpler things, containing the very essence of mathematical thought -- abstractness and generality, must have cost centuries of struggle to devise; yet their originators have vanished leaving not a trace of their lives and personalities. For example, as Bertrand Russell observed, "It must have taken many ages to discover that a brace of pheasants and a couple of days were both instances of the number two." And it took some twenty five centuries of civilization to evolve Russell's own logical definition of "two" or of any cardinal number (reported in the concluding chapter).Again, the conception of a point, which we (erroneously) think we fully understand when we begin school geometry must have come very late in man's career as an artistic, cavepainting animal. Horace Lamb, an English mathematical physicist, would "erect a monument to the unknown mathematical inventor of the mathematical point as the supreme type of that abstraction which has been a necessary condition of scientific work from the beginning."Who, by the way, did invent the mathematical point? In one sense Lamb's forgotten man; in another, Euclid with his definition "a point is that which has no parts and which has no magnitude"; in yet a third sense Descartes with his invention of the "coordinates of a point", until finally in geometry as experts practise it today the mysterious "point" has joined the forgotten man and all his gods in everlasting oblivion, to be replaced by something more usable -- a set of numbers written in a definite order.The last is a modern instance of the abstractness and precision toward which mathematics strives constantly, only to realize when abstractness and precision are attained that a higher degree of abstractness and a sharper precision are demanded for clear understanding. Our own conception of a "point" will no doubt evolve into something yet more abstract. Indeed the "numbers" in terms of which points are described today dissolved about the beginning of this century into the shimmering blue of pure logic, which in its turn seems about to vanish in something rarer and even less substantial.It is not necessarily true then that a step-by-step following of our predecessors is the sure way to understand either their conception of mathematics or our own. Such a retracing of the path that has led up to our present outlook would undoubtedly be of great interest in itself. But it is quicker to glance back over the terrain from the hilltop on which we now stand. The false steps, the crooked trails, and the roads that led nowhere fade out in the distance, and only the broad highways are seen leading straight back to the past, where we lose them in the mists of uncertainty and conjecture. Neither space nor number, nor even time, have the same significance for us that they had for the men whose great figures appear dimly through the mist.A Pythagorean of the sixth century before Christ could intone "Bless us, divine Number, thou who generatest gods and men"; a Kantian of the nineteenth century could refer confidently to "space" as a form of "pure intuition"; a mathematical astronomer could announce a decade ago that the Great Architect of the Universe is a pure mathematician. The most remarkable thing about all of these profound utterances is that human beings no stupider than ourselves once thought they made sense.To a modern mathematician such all-embracing generalities mean less than nothing. Yet in parting with its claim to be the universal generator of gods and men mathematics has gained something more substantial, a faith in itself and in its ability to create human values.Our point of view has changed -- and is still changing. To Descartes' "Give me space and motion and I will give you a world," Einstein today might retort that altogether too much is being asked, and that the demand is in fact meaningless: without a "world" -- matter -- there is neither "space" nor "motion." And to quell the turbulent, muddled mysticism of Leibniz in the seventeenth century, over the mysterious [underoot-1]: "The Divine Spirit found a sublime outlet in that wonder of analysis, the portent of the ideal, that mean between being and not-being, which we call the imaginary [square] root of negative unity," Hamilton in the 1840's constructed a number-couple which any intelligent child can understand and manipulate, and which does for mathematics and science all that the misnamed "imaginary" ever did. The mystical "not-being" of the seventeenth century Leibniz is seen to have a "being" as simple as ABC.Is this a loss? Or does a modern mathematician lose anything of value when he seeks through the postulational method to track down that elusive "feeling" described by Heinrich Hertz, the discoverer of wireless waves: "One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them"?Any competent mathematician will understand Hertz' feeling, but he will also incline to the belief that whereas continents and wireless waves are discovered, dynamos and mathematics are invented and do what we make them do. We can still dream but we need not deliberately court nightmares. If it is true, as Charles Darwin asserted, that "Mathematics seems to endow one with something like a new sense," that sense is the sublimated common sense which the physicist and engineer Lord Kelvin declared mathematics to be.Is it not closer to our own habits of thought to agree temporarily with Galileo that "Nature's great book is written in mathematical symbols" and let it go at that, than to assert with Plato that "God ever geometrizes," or with Jacobi that "God ever arithmetizes"? If we care to inspect the symbols in nature's great book through the critical eyes of modern science we soon perceive that we ourselves did the writing, and that we used the particular script we did because we invented it to fit our own understanding. Some day we may find a more expressive shorthand than mathematics for correlating our experiences of the physical universe -- unless we accept the creed of the scientific mystics that everything is mathematics and is not merely described for our convenience in mathematical language. If "Number rules the universe" as Pythagoras asserted, Number is merely our delegate to the throne, for we rule

Number.When a modern mathematician turns aside for a moment from his symbols to communicate to others the feeling that mathematics inspires in him, he does not echo Pythagoras and Jeans, but he may quote what Bertrand Russell said about a quarter of a century ago: "Mathematics, rightly viewed, possesses not only truth but supreme beauty -- a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."Another, familiar with what has happened to our conception of mathematical "truth" in the years since Russell praised the beauty of mathematics, might refer to the "iron endurance" which some acquire from their attempt to understand what mathematics means, and quote James Thomson's lines (which close this book) in description of Drer's Melencolia (the frontispiece). And if some devotee is reproached for spending his life on what to many may seem the selfish pursuit of a beauty having no immediate reflection in the lives of his fellowmen, he may repeat Poincar's "Mathematics for mathematics' sake. People have been shocked by this formula and yet it is as good as life for life's sake, if life is but misery."To form an estimate of what modern mathematics compared to ancient has accomplished, we may first look at the mere bulk of the work in the period after 1800 compared to that before 1800. The most extensive history of mathematics is that of Moritz Cantor, Geschichte der Mathematik, in three large closely printed volumes (a fourth, by collaborators, supplements the three). The four volumes total about 8600 pages. Only the outline of the development is given by Cantor; there is no attempt to go into details concerning the contributions described, nor are technical terms explained so that an outsider could understand what the whole story is about, and biography is cut to the bone; the history is addressed to those who have some technical training. This history ends with the year 1799 -- just before modern mathematics began to feel its freedom. What if the outline history of mathematics in the nineteenth century alone were attempted on a similar scale? It has been estimated that nineteen or twenty volumes the size of Cantor's would be required to tell the story, say about 17,000 pages. The nineteenth century, on this scale, contributed to mathematical knowledge about five times as much as was done in the whole of preceding history.The beginningless period before 1800 breaks quite sharply into two. The break occurs about the year 1700, and is due mainly to Isaac Newton (1642-1727). Newton's greatest rival in mathematics was Leibniz (1646-1716). According to Leibniz, of all mathematics up to the time of Newton, the more important half is due to Newton. This estimate refers to the power of Newton's general methods rather than to the bulk of his work; the Principia is still rated as the most massive addition to scientific thought ever made by one man.Continuing back into time beyond 1700 we find nothing comparable till we reach the Golden Age of Greece -- a step of nearly 2000 years. Farther back than 600 B.C. we quickly pass into the shadows, coming out into the light again for a moment in ancient Egypt. Finally we arrive at the first great age of mathematics, about 2000 B.C., in the Euphrates Valley.The descendants of the Sumerians in Babylon appear to have been the first "moderns" in mathematics; certainly their attack on algebraic equations is more in the spirit of the algebra we know than anything done by the Greeks in their Golden Age. More important than the technical algebra of these ancient Babylonians is their recognition -- as shown by their work -- of the necessity for proof in mathematics. Until recently it had been supposed that the Greeks were the first to recognize that proof is demanded for mathematical propositions. This was one of the most important steps ever taken by human beings. Unfortunately it was taken so long ago that it led nowhere in particular so far as our own civilization is concerned -- unless the Greeks followed consciously, which they may well have done. They were not particularly generous to their predecessors.Mathematics then has had four great ages: the Babylonian, the Greek, the Newtonian (to give the period around 1700 a name), and the recent, beginning about 1800 and continuing to the present day. Competent judges have called the last the Golden Age of Mathematics.Today mathematical invention (discovery, if you prefer) is going forward more vigorously than ever. The only thing, apparently, that can stop its progress is a general collapse of what we have been pleased to call civilization. If that comes, mathematics may go underground for centuries, as it did after the decline of Babylon; but if history repeats itself, as it is said to do, we may count on the spring bursting forth again, fresher and clearer than ever, long after we and all our stupidities shall have been forgotten.Revue de presseBertrand Russell Professor Bell has done his work well....Any [one] engaged in learning mathematics will profit by reading him, since he humanizes the subject and helps to a realization of the historical environment.The New York Times Extremely harmonious...a first text in the philosophy of mathematics....Bell's style is very enjoyable.Nature Professor E.T. Bell has written a fascinating book. The amount of biographical details and of mathematics that he has compressed into a volume of 600 pages is extraordinary...he carries the reader along; he whets the appetite.